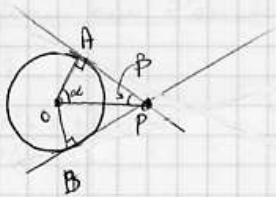


Questi:

1)



$$OA = r \quad OP = 2r$$

$$\operatorname{sen} \beta = \frac{OA}{OP} = \frac{r}{2r} = \frac{1}{2}$$



$$\beta = \frac{\pi}{6} = 30^\circ; \quad \alpha = 90^\circ - \beta = 60^\circ = \frac{\pi}{3}$$

$$S_{\widehat{APB}} = S_{OBPA} - S_{\text{settore, OBA}}$$

$$S_{\text{settore}} = \frac{1}{2} \alpha r^2 = \frac{\pi}{3} r^2$$

$$S_{OBPA} = 2 S_{OAP} = 2 \cdot \frac{1}{2} r \cdot 2r \operatorname{sen} \alpha = 2r^2 \frac{\sqrt{3}}{2} = \sqrt{3} r^2$$

$$S_{\widehat{APB}} = \sqrt{3} r^2 - \frac{\pi}{3} r^2 = r^2 \left(\sqrt{3} - \frac{\pi}{3} \right)$$

2) $\lim_{x \rightarrow +\infty} \frac{(a^2-1)x^3 + (a+1)x^2 - 2x}{x^2 - 3x}$

se $a^2 - 1 = 0 \quad a = \pm 1 \quad f(x) =$

$a = -1 \quad f(x) = \frac{-2x}{x^2 - 3x} \quad \lim_{x \rightarrow +\infty} f(x) = 0$

$a = +1 \quad f(x) = \frac{2x^2 - 2x}{x^2 - 3x} \quad \lim_{x \rightarrow +\infty} f(x) = 2$

se $a^2 - 1 > 0 \quad a < -1 \vee a > 1$

$$\lim_{x \rightarrow +\infty} \frac{(a^2-1)x^3 + (a+1)x^2 - 2x}{x^2 - 3x} = \lim_{x \rightarrow +\infty} \frac{(a^2-1)x^3}{x^2} = +\infty$$

se $a^2 - 1 < 0 \quad -1 < a < 1$

$$\lim_{x \rightarrow +\infty} \frac{(a^2-1)x^3 + (a+1)x^2 - 2x}{x^2 - 3x} = \lim_{x \rightarrow +\infty} \frac{(a^2-1)x^3}{x^2} = -\infty$$

3) Definizioni sul libro.

Procedimento operativo per classificazione e discontinuità di

$$y = \frac{x^2 - x - 6}{x^3 - 4x}$$

$$D \quad x(x^2 - 4) \neq 0$$

$$x \neq 0 \wedge x \neq \pm 2$$

$$\lim_{x \rightarrow 0^\pm} \frac{x^2 - x - 6}{x(x^2 - 4)} = \frac{6}{0^\pm(-4)} = \mp \infty \quad \text{II specie } x=0$$

$$\lim_{x \rightarrow 2^+} \frac{x^2 - x - 6}{x(x-2)(x+2)} = \frac{4 - 2 - 6}{2(0^+)(4)} = \frac{-4}{8 \cdot 0^+} = -\infty \quad \underline{\text{II specie}} \quad x=2$$

$$\lim_{x \rightarrow 2} \frac{x^2 - x - 6}{x(x-2)(x+2)} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 2} \frac{(x-3)(x+2)}{x(x-2)(x+2)} = \frac{-5}{-2(-4)} = -\frac{5}{8}$$

$$N=0 \quad x^2 - x - 6 = 0$$

$$\quad \quad \quad +3$$

$$\quad \quad \quad -2$$

$$x = -2 \quad \underline{\text{III specie}}$$

$$4) \quad f(x) = \log_3(-x) \quad \text{CE} \quad -x > 0 \quad x < 0$$

$$g(x) = \log_3(9-x) \quad \text{CE} \quad 9-x > 0 \quad x < 9$$

$$|f(x) - g(x)| = |f(x) + g(x)| \quad \text{CE} \quad \begin{cases} x < 0 \\ x < 9 \end{cases} \Rightarrow \boxed{x < 0} \quad \text{CE}$$

$$|\log_3(-x) - \log_3(9-x)| = |\log_3(-x) + \log_3(9-x)|$$

$$|\log_3 \frac{-x}{9-x}| = |\log_3 -x(9-x)|$$

$$\log_3 \frac{-x}{9-x} = \pm \log_3 [-x(9-x)]$$

$$\textcircled{1}^+ \quad \frac{-x}{9-x} = -x(9-x) \quad x \neq 9$$

$$\frac{-x + x(9-x)^2}{9-x} = 0 \quad x[-1 + (9-x)^2] = 0 \quad x = 0 \quad \underline{\text{non see.}}$$

$$(9-x)^2 = 1$$

$$9-x = \pm 1 \quad x_1 = 8 \quad \underline{\text{non see.}}$$

$$x_2 = 10 \quad \underline{\text{non see.}}$$

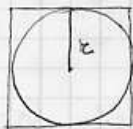
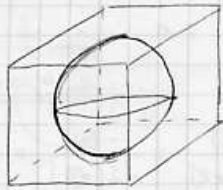
$$\textcircled{2}^- \quad \log_3 \frac{-x}{9-x} = -\log_3(-x(9-x))$$

$$\log_3 \frac{-x}{9-x} + \log_3[-x(9-x)] = 0 \quad \frac{-x}{9-x} \cdot (-x(9-x)) = 1 \quad x \neq 9$$

$$x^2 = 1 \quad x = +1 \quad \underline{\text{non see. per il CE}}$$

$$\boxed{x = -1 \quad \text{see.}}$$

5)



$$l = 2r$$

$$V_{\text{sfera}} = \frac{4}{3} \pi r^3$$

$$V_{\text{cubo}} = l^3 = 8r^3$$

$$\frac{V_{\text{sfera}}}{V_{\text{cubo}}} = \frac{\frac{4}{3} \pi r^3}{8r^3} = \frac{\pi}{6}$$

6) Risposta esatta e.

$$y = \frac{x^4}{x^2 - 4}$$

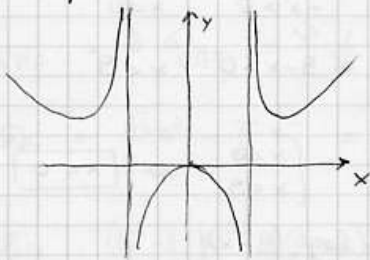
è pari

ha as. verticali $x = \pm 2$

non ha asintoti orizz.

non ha as. obliqui

$$0(0;0) \in \gamma$$



a. $y = x(x^2 - 4)$

D = R (no)

b. $y = \frac{x^3}{x^2 - 4}$

è dispari (no)

d. $y = \frac{4x^2}{x^2 - 4}$

ha as. orizzontale $y = 4$ (no)

c. $y = \frac{x^2 - 4}{x^2}$

D: $x \neq 0$ (no)

$$4) \lim_{x \rightarrow 0} \frac{e^{x^3} - 1}{x \sec^2 x} = \lim_{x \rightarrow 0} \frac{e^{x^3} - 1}{x} \cdot \frac{x^2}{x^2} \cdot \frac{1}{\sec^2 x} = \lim_{x \rightarrow 0} \frac{e^{x^3} - 1}{x^3} \cdot \frac{x^2}{\sec^2 x} = 1$$

9) $y = \frac{\sqrt{2 \sin(2x) - \sqrt{3}}}{\log \cos x}$

con $0 \leq x \leq 2\pi$

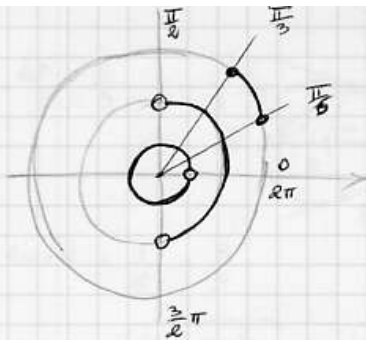
$$\text{e} \begin{cases} \log \cos x \neq 0 \\ \cos x > 0 \\ 2 \sin 2x - \sqrt{3} \geq 0 \end{cases}$$

$$\begin{cases} \cos x \neq 1 \\ \cos x > 0 \\ \sin 2x \geq \frac{\sqrt{3}}{2} \end{cases}$$



$$\begin{cases} x \neq 0; 2\pi \\ \frac{3}{4}\pi < x < 2\pi \vee 0 < x < \frac{\pi}{2} \end{cases}$$

$$\frac{\pi}{3} \leq 2x \leq \frac{5}{6}\pi \Rightarrow \frac{\pi}{6} \leq x \leq \frac{\pi}{3}$$



$$\frac{\pi}{6} \leq x \leq \frac{\pi}{3} \quad \text{CF}$$

$$a) \quad s_1 = 3t^2 - 2t \quad v_1 = 6t - 2$$

$$s_2 = -3 + \frac{1}{2}t + t^2 \quad v_2 = \frac{1}{2} + 2t$$

Nell'istante t in cui si incontrano

$$s_1 = s_2$$

$$3t^2 - 2t = -3 + \frac{1}{2}t + t^2$$

$$2t^2 - 2t - \frac{1}{2}t - 3 = 0$$

$$2t^2 - \frac{5}{2}t - 3 = 0$$

$$4t^2 - 5t - 6 = 0 \quad t_{1,2} = \frac{5 \pm \sqrt{25 + 96}}{8} = \frac{5 \pm 11}{8}$$

$$t_1 = \frac{16}{8} = 2s$$

$$t_2 = -\frac{6}{8} \quad \text{non see.}$$

$$v_1 = 6 \cdot 2 - 2 = 10 \frac{\text{m}}{\text{s}}$$

$$v_2 = \frac{1}{2} + 2 \cdot 2 = \frac{9}{2} = 4,5 \frac{\text{m}}{\text{s}}$$

$$10) \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad f(x) = \sqrt{x^3}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{(x+h)^3} - \sqrt{x^3}}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h [\sqrt{(x+h)^3} + \sqrt{x^3}]} =$$

$$= \lim_{h \rightarrow 0} \frac{x^3 + h^3 + 3x^2h + 3xh^2 - x^3}{h [\sqrt{(x+h)^3} + \sqrt{x^3}]} = \lim$$

$$= \lim_{h \rightarrow 0} \frac{h^2 + 3x^2h + 3xh^2}{\sqrt{(x+h)^3} + \sqrt{x^3}} = \frac{3x^2}{2\sqrt{x^3}} = \frac{3x^2}{2x\sqrt{x}} = \frac{3\sqrt{x}}{2}$$

$$\text{Brefatti} \quad f'(x) = D x^{\frac{3}{2}} = \frac{3}{2} x^{\frac{3}{2}-1} = \frac{3}{2} x^{\frac{1}{2}} = \frac{3}{2} \sqrt{x}$$

The End