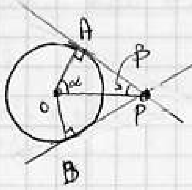
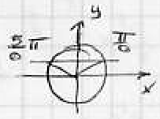
Quesiti

1)



$\overline{OA} = r \qquad \overline{OP} = 2r$

$\operatorname{sen}\beta = \dfrac{\overline{OA}}{\overline{OP}} = \dfrac{r}{2r} = \dfrac{1}{2}$



$\beta = \frac{\pi}{6} = 30^\circ; \quad \alpha = 90^\circ - \beta = 60^\circ = \frac{\pi}{3}$

$S_{\widehat{APB}} = S_{OBPA} - S_{\text{settore}, OBA}$

$S_{\text{settore}} = \frac{1}{2} 2\alpha\, r^2 = \frac{\pi}{3} r^2$

$S_{OBPA} = 2\, S_{OAP} = \cancel{2}\, \frac{1}{\cancel{2}}\, r \cdot 2r \operatorname{sen}\alpha = \cancel{2} r^2 \frac{\sqrt{3}}{\cancel{2}} = \sqrt{3}\, r^2$

$S_{\widehat{APB}} = \sqrt{3} r^2 - \frac{\pi}{3} r^2 = r^2\left(\sqrt{3} - \frac{\pi}{3}\right)$

2) $\displaystyle\lim_{x \to +\infty} \frac{(a^2-1)x^3 + (a+1)x^2 - 2x}{x^2 - 3x}$

se $a^2 - 1 = 0 \qquad a = \pm 1 \qquad f(x) =$

$a = -1 \qquad f(x) = \dfrac{-2x}{x^2-3x} \qquad \displaystyle\lim_{x\to+\infty} f(x) = 0$

$a = +1 \qquad f(x) = \dfrac{2x^2-2x}{x^2-3x} \qquad \displaystyle\lim_{x\to+\infty} f(x) = 2$

se $a^2 - 1 > 0 \qquad a < -1 \vee a > 1$

$\displaystyle\lim_{x\to+\infty} \frac{(a^2-1)x^3 + (a+1)x^2 - 2x}{x^2-3x} = \lim_{x\to+\infty} \frac{(a^2-1)x^{\cancel{3}}}{\cancel{x^2}} = +\infty$

se $a^2 - 1 < 0 \qquad -1 < a < 1$

$\displaystyle\lim_{x\to+\infty} \frac{(a^2-1)x^3 + (a+1)x^2 - 2x}{x^2-3x} = \lim_{x\to+\infty} \frac{(a^2-1)x^{\cancel{3}}}{\cancel{x^2}} = -\infty$

3) Definizioni sul libro.

Procedimento operativo per classificare le discontinuità di

$y = \dfrac{x^2 - x - 6}{x^3 - 4x} \qquad D \quad x(x^2-4) \neq 0$

$x \neq 0 \wedge x \neq \pm 2$

$\displaystyle\lim_{x\to 0^\pm} \frac{x^2-x-6}{x(x^2-4)} = \frac{6}{0^\pm(-4)} = \mp\infty \qquad \underline{\text{II specie} \quad x=0}$

$$\lim_{x \to 2^{\pm}} \frac{x^2-x-6}{x(x-2)(x+2)} = \frac{4-2-6}{2(0^{\pm})4} = \frac{-4}{8\cdot 0^{\pm}} = \mp\infty$$

$\underline{x = 2}$

$\underline{\text{II speţă}}$

$$\lim_{x \to -2} \frac{x^2-x-6}{x(x-2)(x+2)} = \left[\frac{0}{0}\right] = \lim_{x\to -2} \frac{(x-3)\cancel{(x+2)}}{x(x-2)\cancel{(x+2)}} = \frac{-5}{-2(-4)} = -\frac{5}{8}$$

$\underline{x=-2 \quad \text{III speţă}}$

$N=0 \quad x^2-x-6=0$

$+3$

-2

4) $f(x) = \log_3(-x) \qquad CE \quad -x>0 \quad x<0$

$g(x) = \log_3(9-x) \qquad CE \quad 9-x>0 \quad x<9$

$$|f(x)-g(x)| = |f(x)+g(x)| \qquad CE \quad \begin{cases} x<0 \\ x<9 \end{cases} \Rightarrow \boxed{x<0} \; CE$$

$$|\log_3(-x) - \log_3(9-x)| = |\log_3(-x) + \log_3(9-x)|$$

$$\left|\log_3 \frac{-x}{9-x}\right| = |\log_3 -x(9-x)|$$

$$\log_3 \frac{-x}{9-x} = \pm \log_3 [-x(9-x)]$$

① $^+$

$$\frac{-x}{9-x} = -x(9-x) \qquad x \neq 9$$

$$\frac{-x+x(9-x)^2}{\cancel{9-x}} = 0 \qquad x[-1+(9-x)^2]=0 \qquad x=0 \text{ nu conv.}$$

$(9-x)^2 = 1$

$9-x = \pm 1 \qquad x_1 = 8 \quad \underline{\text{nu conv.}}$

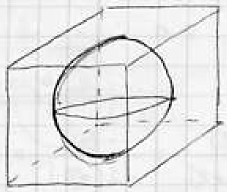
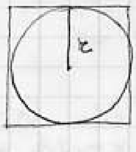
$x_2 = 10 \quad \underline{\text{nu conv.}}$

② $^-$ $\log_3 \frac{-x}{9-x} = -\log_3(-x(9-x))$

$$\log_3 \frac{-x}{9-x} + \log_3[-x(9-x)] = 0 \qquad \frac{-x}{\cancel{9-x}}\cdot(-x\cancel{(9-x)}) = 1 \qquad x\neq 9$$

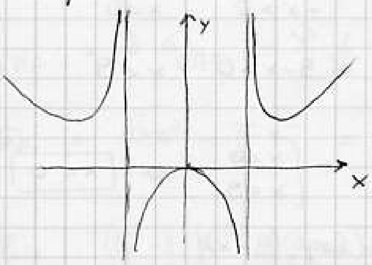
$x^2 = 1 \qquad x = +1 \quad \underline{\text{nu conv. față de CE}}$

$$\boxed{x = -1 \quad \text{conv.}}$$

5)   $\ell = 2r$

$V_{sfera} = \frac{4}{3}\pi r^3$ $\qquad V_{cubo} = \ell^3 = 8r^3$ $\qquad \frac{V_{sfera}}{V_{cubo}} = \frac{\frac{\cancel{4}}{3}\pi \cancel{r^3}}{\cancel{8}\cancel{r^3}} = \frac{\pi}{6}$

6) risposta esatta e. $\quad y = \frac{x^4}{x^2-4}$



è pari

ha as. verticali $x = \pm 2$

non ha asintoti orizz.

non ha as. obliqui

$O(0;0) \in \gamma$

a. $\quad y = x(x^2-4) \qquad D = \mathbb{R}$ no

b. $\quad y = \frac{x^3}{x^2-4} \qquad$ è dispari no

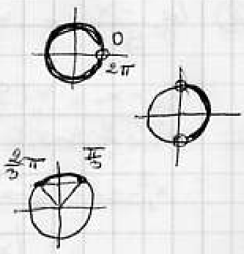
d. $\quad y = \frac{4x^2}{x^2-4} \qquad$ ha as. orizzontale $y = 4$ no

e. $\quad y = \frac{x^2-4}{x^2} \qquad D: \; x \neq 0$ no

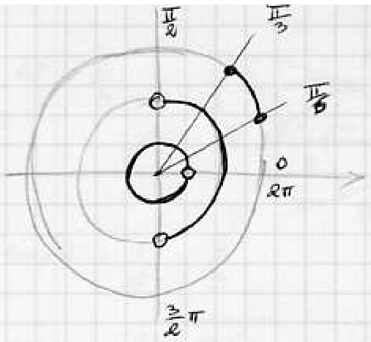
7) $$\lim_{x\to 0} \frac{e^{x^3}-1}{x \operatorname{sen}^2 x} = \lim_{x\to 0} \frac{e^{x^3}-1}{x} \frac{x^2}{x^2} \frac{1}{\operatorname{sen}^2 x} = \lim_{x\to 0} \frac{e^{x^3}-1}{x^3} \cdot \frac{x^2}{\operatorname{sen}^2 x} = 1$$

9) $$y = \frac{\sqrt{2\operatorname{sen}(2x) - \sqrt{3}}}{\log \cos x} \qquad \text{con} \qquad 0 \le x \le 2\pi$$

C.E. $\begin{cases} \log \cos x \neq 0 \\ \cos x > 0 \\ 2 \operatorname{sen} 2x - \sqrt{3} \ge 0 \end{cases}$ $\qquad \begin{cases} \cos x \neq 1 \\ \cos x > 0 \\ \operatorname{sen} 2x \ge \frac{\sqrt{3}}{2} \end{cases}$



$$\begin{cases} x \neq 0;\, 2\pi \\ \frac{3}{2}\pi < x < 2\pi \;\vee\; 0 < x < \frac{\pi}{2} \\ \frac{\pi}{3} \le 2x \le \frac{2}{3}\pi \quad \Rightarrow \quad \frac{\pi}{6} \le x \le \frac{\pi}{3} \end{cases}$$



$\frac{\pi}{6} \le x \le \frac{\pi}{3}$ CE

9) $s_1 = 3t^2 - 2t$ $\qquad v_1 = 6t - 2$

$s_2 = 3 + \frac{1}{2}t + t^2$ $\qquad v_2 = \frac{1}{2} + 2t$

Nell'istante t in cui si incontrano

$s_1 = s_2$ $\qquad 3t^2 - 2t = 3 + \frac{1}{2}t + t^2$

$2t^2 - 2t - \frac{1}{2}t - 3 = 0$

$2t^2 - \frac{5}{2}t - 3 = 0$

$4t^2 - 5t - 6 = 0 \qquad t_{1,2} = \frac{5 \pm \sqrt{25+96}}{8} = \frac{5 \pm 11}{8}$

$t_1 = \frac{16}{8} = 2s$

$t_2 = -\frac{6}{8}$ non acc.

$v_1 = 6 \cdot 2 - 2 = 10 \frac{m}{s}$

$v_2 = \frac{1}{2} + 2 \cdot 2 = \frac{9}{2} = 4,5 \frac{m}{s}$

10) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad f(x) = \sqrt{x^3}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{(x+h)^3} - \sqrt{x^3}}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h\left[\sqrt{(x+h)^3} + \sqrt{x^3}\right]} =$$

$\left[\frac{0}{0}\right]$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + h^3 + 3x^2h + 3xh^2 - \cancel{x^3}}{h\left[\sqrt{(x+h)^3} + \sqrt{x^3}\right]} = \lim$$

$$= \lim_{h \to 0} \frac{h^2 + 3x^2 + 3xh}{\sqrt{(x+h)^3} + \sqrt{x^3}} = \frac{3x^2}{2\sqrt{x^3}} = \frac{3x^{\cancel{2}}}{2\cancel{x}\sqrt{x}} = \frac{3\cancel{x}\sqrt{x}}{2\cancel{x}} = \frac{3}{2}\sqrt{x}$$

Infatti $f'(x) = D\,x^{\frac{3}{2}} = \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$.

The End