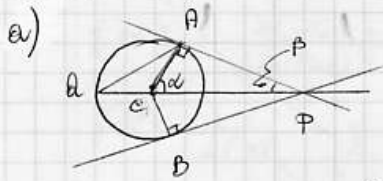


Problema 2



$\overline{OP} = x$ Limitazioni geom. $x > 1$
 $\overline{OA} = r = 1$
 $\overline{AP} = \sqrt{\overline{OP}^2 - \overline{OA}^2} = \sqrt{x^2 - 1}$

PO bisettrice di $\hat{BPA} \Rightarrow \hat{CBP} \cong \hat{CAP}$ per il 1° criterio di congruenza.

$\cos \alpha = \frac{\overline{OA}}{\overline{OP}} = \frac{1}{x}$ $\text{sen} \beta = \cos \alpha$

$\text{sen} \alpha = \frac{\overline{AP}}{\overline{OP}} = \frac{\sqrt{x^2 - 1}}{x}$ $\cos \beta = \text{sen} \alpha$

b) $\hat{OEA} = \alpha'$ $\alpha' = 180^\circ - \alpha$ $\cos \alpha' = \cos(180^\circ - \alpha) = -\cos \alpha = -\frac{1}{x}$

$\overline{AA'}^2 = \overline{OA}^2 + \overline{OA'}^2 - 2 \overline{OA} \cdot \overline{OA'} \cos \alpha' =$
 $= 1^2 + 1^2 - 2 \cdot 1 \cdot 1 \cdot \left(-\frac{1}{x}\right) = 2 + \frac{2}{x} = \frac{2(x+1)}{x}$

$f(x) = \frac{1}{\overline{AP}^2} + \frac{1}{\overline{AA'}^2} = \frac{1}{x^2 - 1} + \frac{x}{2(x+1)} = \frac{2 + (x-1)}{2(x^2-1)} = \frac{x^2 - x + 2}{2(x^2-1)}$

c) Grafico $f(x) = \frac{x^2 - x + 2}{2(x^2 - 1)}$ $D: x^2 - 1 \neq 0 \quad x \neq \pm 1$

$D = (-\infty; -1) \cup (1; +\infty)$

in pari, in dispari

Zeri $\begin{cases} x=0 \\ y=-1 \end{cases}$ $\begin{cases} y=0 \\ \frac{x^2 - x + 2}{2(x^2 - 1)} = 0 \end{cases}$ $\begin{cases} - \\ x^2 - x + 2 = 0 \end{cases}$
 $A(0; -1)$ $\Delta = 1 - 8 < 0$ non esistono intersezioni con l'asse x

segno $N > 0$ $x^2 - x + 2 > 0 \quad \forall x \in D$
 $D > 0$ $2(x^2 - 1) > 0 \quad |x < -1 \vee x > 1| \quad \text{IP}$

limiti e asintoti

$\lim_{x \rightarrow \pm \infty} \frac{x^2 - x + 2}{2(x^2 - 1)} = \lim_{x \rightarrow \pm \infty} \frac{x^2 \left(1 - \frac{1}{x} + \frac{2}{x^2}\right)}{x^2 \left(2 - \frac{1}{x^2}\right)} = \frac{1}{2}$ $y = \frac{1}{2}$ as. orizz.

$\lim_{x \rightarrow -1^\pm} \frac{x^2 - x + 2}{2(x+1)(x-1)} = \frac{1+1+2}{2 \cdot 0^\pm \cdot (-2)} = \mp \infty$
 $\lim_{x \rightarrow 1^\pm} \frac{x^2 - x + 2}{2(x+1)(x-1)} = \frac{1-1+2}{2 \cdot 2 \cdot 0^\pm} = \pm \infty$ $\Rightarrow x = \pm 1$ as. verticali

intersezione con l'elemento $y = \frac{1}{2}$

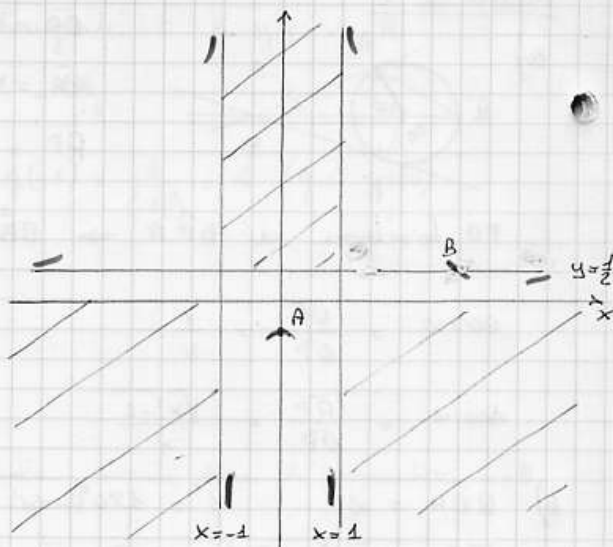
$$\begin{cases} y = \frac{1}{2} \\ y = \frac{x^2 - x + 2}{2(x^2 - 1)} \end{cases}$$

$$(*) \frac{x^2 - x + 2}{2(x^2 - 1)} = \frac{1}{2}$$

$$(c) \frac{x^2 - x + 2 - (x^2 - 1)}{x^2 - 1} = 0$$

$$\frac{x^2 - x + 2 - x^2 + 1}{x^2 - 1} = 0 \quad x \neq \pm 1$$

$$x - 3 = 0 \quad \begin{cases} x = 3 \\ y = \frac{1}{2} \end{cases} \quad B(3; \frac{1}{2})$$



Studio la derivata prima

$$y' = \frac{(2x-1)(2x^2-2) - (x^2-x+2)4x}{4(x^2-1)^2} = \frac{4x^3 - 4x - 2x^2 + 2 - 4x^3 + 4x^2 - 8x}{4(x^2-1)^2} = \frac{2x^2 - 12x + 2}{4(x^2-1)^2} = \frac{x^2 - 6x + 1}{2(x^2-1)^2} \quad D' \quad x \neq \pm 1$$

$$y' = 0 \quad x^2 - 6x + 1 = 0 \quad x_{1,2} = \frac{3 \pm \sqrt{9-1}}{2} = 3 \pm 2\sqrt{2}$$

$$y' > 0 \quad N > 0 \quad x < 3 - 2\sqrt{2} \quad \vee \quad x > 3 + 2\sqrt{2}$$

$$D > 0 \quad (x^2 - 1)^2 > 0 \quad x \neq \pm 1$$

D:	-1	$3-2\sqrt{2}$	1	$3+2\sqrt{2}$			
N	+	+0	-	-	0	+	
D	+	0+	+0+	+	0	+	
$f'(x) = N/D$	+	+ 0	-	- 0	-	0	+
$f(x)$	↗	↗	↘	↘	↗		
		M		Q			

tabella di monotonia

M massimo stazionario

$M(3-2\sqrt{2})$;

Q minimo stazionario

$Q(3+2\sqrt{2})$;

d) Tangente in $A(0; -1)$

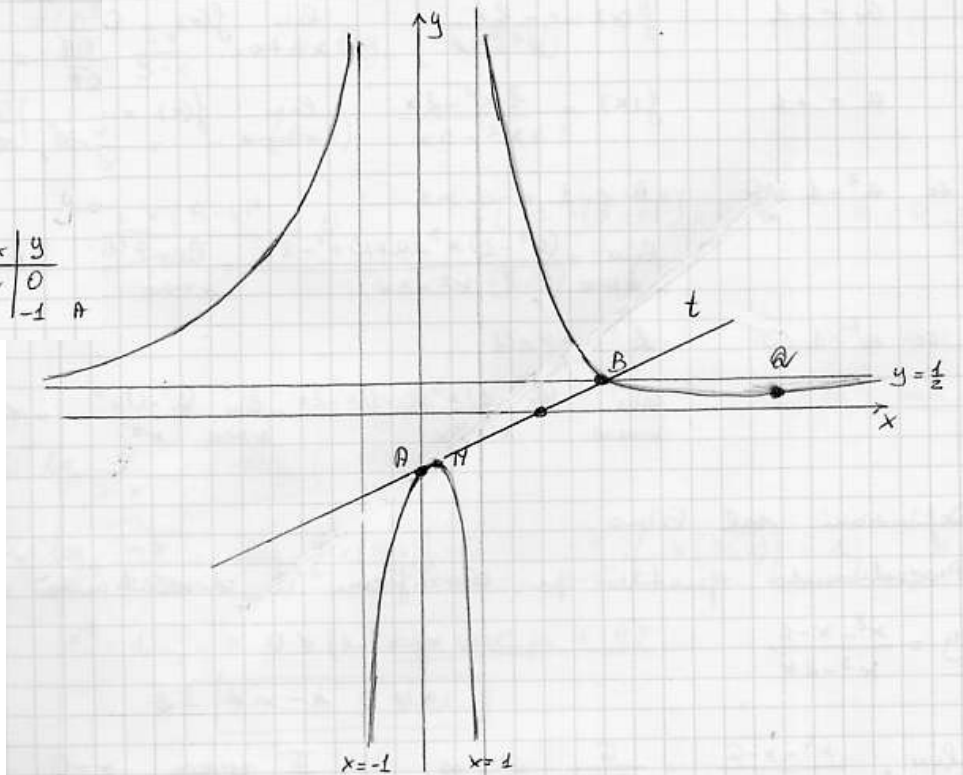
$$f'(x) = \frac{x^2 - 6x + 1}{2(x^2 - 1)^2}$$

$$f'(0) = \frac{1}{2(-1)^2} = \frac{1}{2}$$

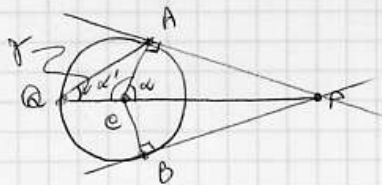
t: $y + 1 = \frac{1}{2}x$

$$y = \frac{1}{2}x - 1$$

x	y
2	0
0	-1



e) $\overline{CP} = \frac{5}{3}$



$$\cos \alpha = \frac{1}{x} = \frac{3}{5}$$

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{4}{5} \cdot \frac{5}{3} = \frac{4}{3}$$

$$\operatorname{cotg} \alpha = \frac{1}{\operatorname{tg} \alpha} = \frac{3}{4}$$

$$\boxed{\widehat{QCA}} \quad \alpha' = \widehat{QCA} = 180^\circ - \alpha$$

$$\cos \alpha' = \cos(180^\circ - \alpha) = -\cos \alpha = -\frac{3}{5}$$

$$\sin \alpha' = \sqrt{1 - \cos^2 \alpha'} = \frac{4}{5}$$

$$\operatorname{tg} \alpha' = -\frac{4}{3} \quad \operatorname{cotg} \alpha' = -\frac{3}{4}$$

$$\gamma = 90^\circ - \frac{\alpha'}{2}$$

$$\begin{aligned} \cos \gamma &= \cos\left(90^\circ - \frac{\alpha'}{2}\right) = \sin \frac{\alpha'}{2} = \pm \sqrt{\frac{1 - \cos \alpha'}{2}} \\ &= \sqrt{\left(1 + \frac{3}{5}\right) \frac{1}{2}} = \sqrt{\frac{8}{10}} = \sqrt{\frac{4}{5}} = \frac{2\sqrt{5}}{5} \end{aligned}$$

$$\sin \gamma = \pm \sqrt{1 - \cos^2 \gamma} = \sqrt{1 - \frac{4}{5}} = \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5}$$

$$\operatorname{tg} \gamma = \frac{\sin \gamma}{\cos \gamma} = \frac{\sqrt{5}}{5} \cdot \frac{5}{2\sqrt{5}} = \frac{1}{2}$$

$$\operatorname{cotg} \gamma = \frac{1}{\operatorname{tg} \gamma} = 2$$