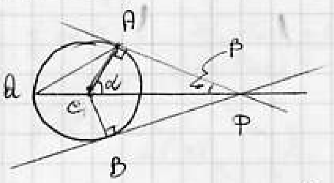
Problema 2

a)



$\overline{CP} = x$ limitazioni geom. $x > 1$

$\overline{CA} = r = 1$

$\overline{AP} = \sqrt{\overline{OP}^2 - \overline{OA}^2} = \sqrt{x^2-1}$

PC bisettrice di $B\hat{P}A \Rightarrow C\hat{B}P \cong C\hat{A}P$ per il 1° criterio di congruenza -

$\cos\alpha = \frac{\overline{CA}}{\overline{CP}} = \frac{1}{x}$ $\quad \operatorname{sen}\beta = \cos\alpha$

$\operatorname{sen}\alpha = \frac{\overline{AP}}{\overline{CP}} = \frac{\sqrt{x^2-1}}{x}$ $\quad \cos\beta = \operatorname{sen}\alpha$

b) $Q\hat{C}A = \alpha'$ $\quad \alpha' = 180° - \alpha$ $\quad \cos\alpha' = \cos(180° - \alpha) = -\cos\alpha = -\frac{1}{x}$

$$\overline{AQ}^2 = \overline{CQ}^2 + \overline{CA}^2 - 2\,\overline{OQ}\cdot\overline{CA}\cos\alpha' =$$

$$= 1^2 + 1^2 - 2\cdot 1\cdot 1\cdot\left(-\frac{1}{x}\right) = 2 + \frac{2}{x} = \frac{2(x+1)}{x}$$

$$f(x) = \frac{1}{\overline{AP}^2} + \frac{1}{\overline{AQ}^2} = \frac{1}{x^2-1} + \frac{x}{2(x+1)} = \frac{2 + x(x-1)}{2(x^2-1)} = \frac{x^2-x+2}{2(x^2-1)}$$

c) Grafico $\quad f(x) = \frac{x^2-x+2}{2(x^2-1)}$ $\quad D:\ x^2-1 \neq 0 \quad x \neq \pm 1$

$D = (-\infty; -1) \cup (-1; 1) \cup (1; +\infty)$

né pari, né dispari

Zeri $\begin{cases} x = 0 \\ y = -1 \end{cases}$ $\quad A(0; -1)$ $\qquad \begin{cases} y = 0 \\ \frac{x^2-x+2}{2(x^2-1)} = 0 \end{cases}$ $\qquad \begin{cases} - \\ x^2-x+2 = 0 \end{cases}$

$\Delta = 1 - 8 < 0$ non ci sono inters. con l'asse x

segno $\quad N > 0 \quad x^2-x+2 > 0 \quad \forall x \in D$

$D > 0 \quad 2(x^2-1) > 0 \quad \boxed{x < -1 \lor x > 1 \quad IP}$

limiti e asintoti

$$\lim_{x\to\pm\infty} \frac{x^2-x+2}{2(x^2-1)} = \lim_{x\to\pm\infty} \frac{x^2\left(1 - \frac{1}{x} + \frac{2}{x^2}\right)}{x^2\left(2 - \frac{1}{x^2}\right)} = \frac{1}{2} \qquad y = \frac{1}{2} \text{ as. orizz.}$$

$$\lim_{x\to -1^\pm} \frac{x^2-x+2}{2(x+1)(x-1)} = \frac{1+1+2}{2\cdot 0^\pm\cdot(-2)} = \mp\infty$$

$$\lim_{x\to 1^\pm} \frac{x^2-x+2}{2(x+1)(x-1)} = \frac{1-1+2}{2\cdot 2\cdot 0^\pm} = \pm\infty$$

$\Rightarrow x = \pm 1$ as. verticali

intersezioni con l'asintoto $y=\frac{1}{2}$

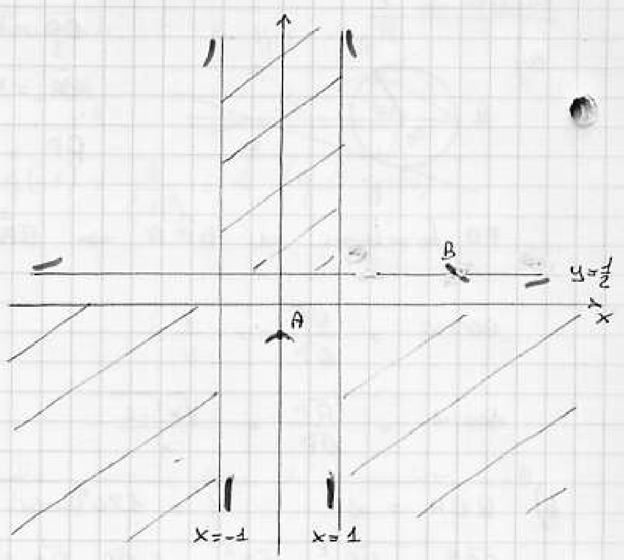
$$\begin{cases} y=\frac{1}{2} \\ y=\frac{x^2-x+2}{2(x^2-1)} \end{cases}$$

$$(\cdot)\begin{cases} - \\ \frac{x^2-x+2}{\cancel{2}(x^2-1)}=\frac{1}{\cancel{2}} \end{cases}$$

$$(\cdot)\ \frac{x^2-x+2-(x^2-1)}{x^2-1}=0$$

$$\frac{\cancel{x^2}-x+2-\cancel{x^2}+1}{\cancel{x^2-1}}=0 \qquad x\neq\pm1$$

$$x-3=0 \qquad \begin{cases} x=3 \\ y=\frac{1}{2} \end{cases} \qquad B\left(3;\frac{1}{2}\right)$$



Studio la derivata prima

$$y'=\frac{(2x-1)(2x^2-2)-(x^2-x+2)4x}{\cancel{4}(x^2-1)^2}=\frac{4x^3-4x-2x^2+2-4x^3+4x^2-8x}{\cancel{4}(x^2-1)^2}=$$

$$=\frac{2x^2-12x+2}{\cancel{4}(x^2-1)^2}=\frac{x^2-6x+1}{2(x^2-1)^2} \qquad D'\quad x\neq\pm1$$

$$y'=0 \qquad x^2-6x+1=0 \qquad x_{1/2}=3\pm\sqrt{9-1}=3\pm2\sqrt{2}$$

$y'>0$ $\quad N>0 \quad x<3-2\sqrt{2} \ \vee\ x>3+2\sqrt{2}$

$\qquad D>0 \quad (x^2-1)^2>0 \quad x\neq\pm1$

D:		-1		$3-2\sqrt{2}$		1		$3+2\sqrt{2}$	
N	+		+	0	−		−	0	+
D	+	0	+		+	0	+		+
$f'(x)=N/D$	+	∄	+	0	−	∄	−	0	+
$f(x)$	↗		↗	M	↘		↘	Q	↗

tabella di monotonia

M massimo stazionario

$M(3-2\sqrt{2}\ ;$

Q minimo stazionario

$Q(3+2\sqrt{2}\ ;$

d) Tangente in $A(0;-1)$

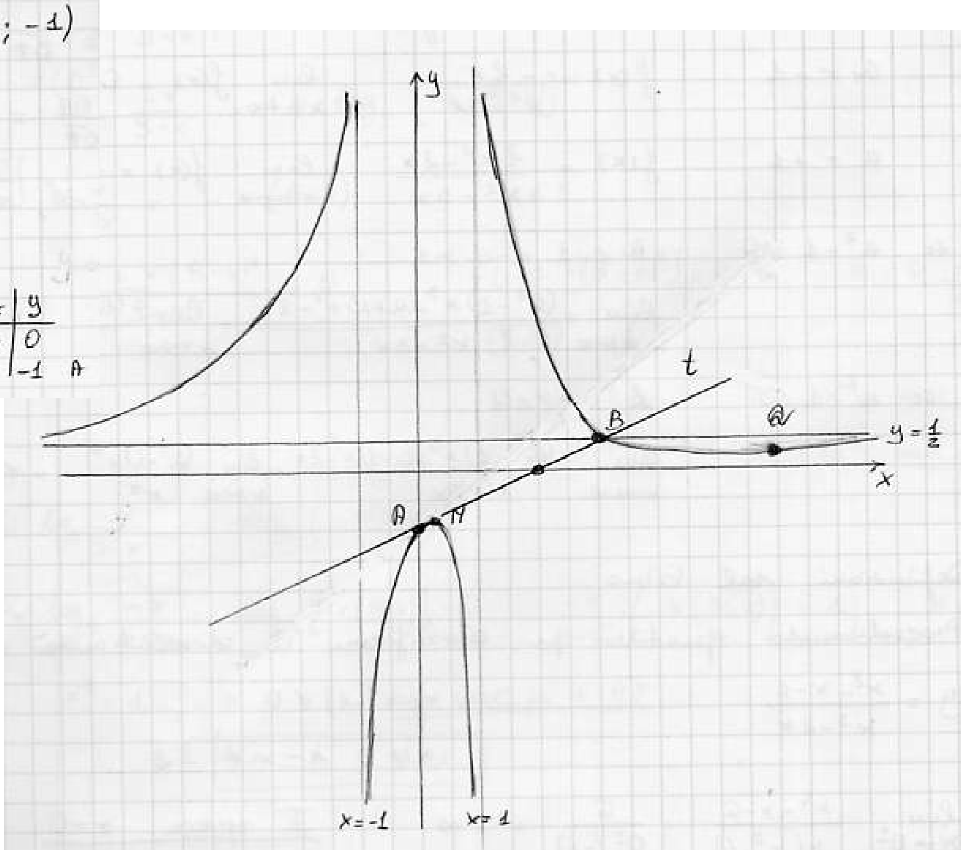
$$f'(x) = \frac{x^2-6x+1}{2(x^2-1)^2}$$

$$f'(0) = \frac{1}{2(-1)^2} = \frac{1}{2}$$

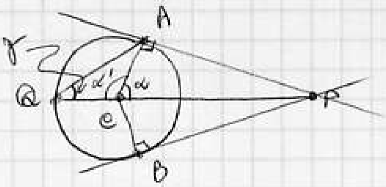
t: $\quad y+1 = \frac{1}{2}x$

$$y = \frac{1}{2}x - 1$$

x	y	
2	0	
0	-1	A



e) $\overline{CP} = \frac{5}{3}$



$$\cos\alpha = \frac{1}{x} = \frac{3}{5}$$

$$\operatorname{sen}\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$$

$\Rightarrow$

$$\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{4}{5}\cdot\frac{5}{3} = \frac{4}{3}$$

$$\operatorname{cotg}\alpha = \frac{1}{\operatorname{tg}\alpha} = \frac{3}{4}$$

$\boxed{Q\hat{C}A}$

$$\alpha' = Q\hat{C}A = 180° - \alpha$$

$$\cos\alpha' = \cos(180°-\alpha) = -\cos\alpha = -\frac{3}{5}$$

$$\operatorname{sen}\alpha' = \sqrt{1-\cos^2\alpha'} = \frac{4}{5}$$

$$\operatorname{tg}\alpha' = -\frac{4}{3} \qquad \operatorname{cotg}\alpha' = -\frac{3}{4}$$

$$\gamma = 90° - \frac{\alpha'}{2}$$

$$\cos\gamma = \cos\left(90° - \frac{\alpha'}{2}\right) = \operatorname{sen}\frac{\alpha'}{2} = +\sqrt{\frac{1-\cos\alpha'}{2}} =$$

$$= \sqrt{\left(1+\frac{3}{5}\right)\frac{1}{2}} = \sqrt{\frac{8}{10}} = \sqrt{\frac{4}{5}} = \frac{2\sqrt{5}}{5}$$

$$\operatorname{sen}\gamma = +\sqrt{1-\cos^2\gamma} = \sqrt{1-\frac{4}{5}} = \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5}$$

$$\operatorname{tg}\gamma = \frac{\operatorname{sen}\gamma}{\cos\gamma} = \frac{\sqrt{5}}{5}\,\frac{5}{2\sqrt{5}} = \frac{1}{2}$$

$$\operatorname{cotg}\gamma = \frac{1}{\operatorname{tg}\gamma} = 2$$